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CP PUBLICATION

SHORT TRICKS

Mathematics

for JEE (Main & Adv)

Sunil Gupta
Career Point Editorial

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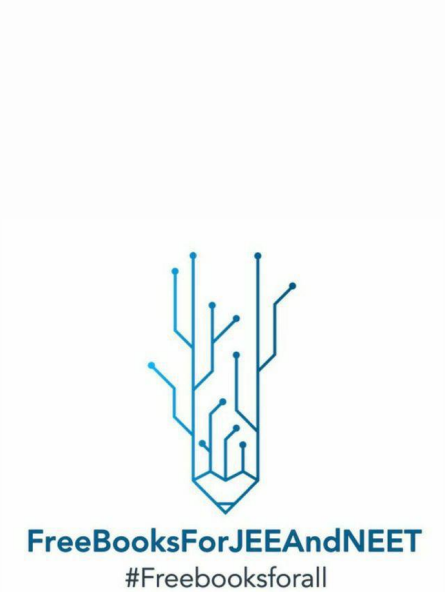
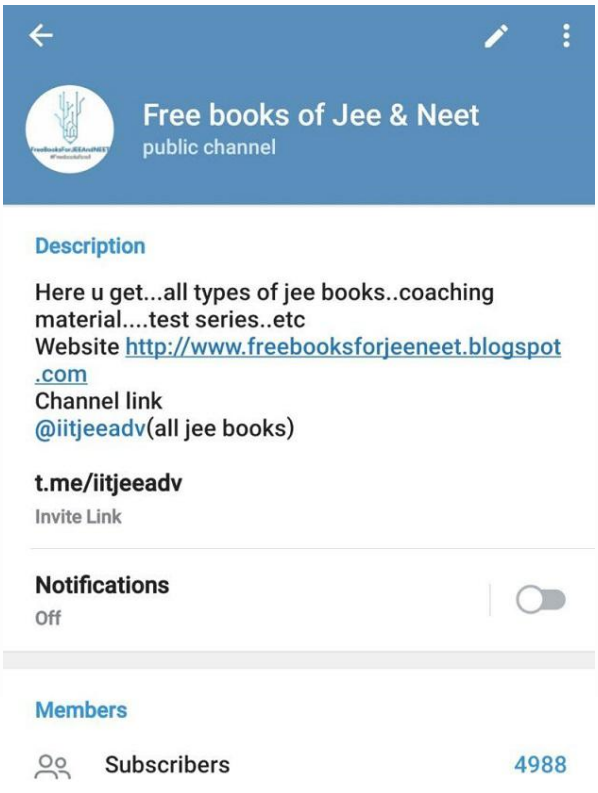
SHORT TRICKS
Mathematics
for JEE (Main & Advanced)





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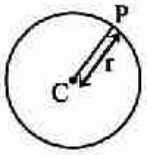
11

Circle

KEY CONCEPTS

A circle is the locus of a point which moves in a plane in such a way that its distance from a fixed point remains constant.



1. General Equation of Circle

The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$

Centre of circle is $(-g, -f)$

i.e. $\left(-\frac{1}{2}\text{coefficient of } x, -\frac{1}{2}\text{coefficient of } y\right)$ and radius of circle is $r = \sqrt{g^2 + f^2 - c}$

(i) If $(g^2 + f^2 - c) > 0$, then the circle is a real circle.

(ii) If $(g^2 + f^2 - c) = 0$, then the circle is a point circle .

(iii) If $(g^2 + f^2 - c) < 0$, then the circle is an imaginary circle with real centre.

(iv) The general equation of second degree $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ represents a circle if

(a) $\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0.$

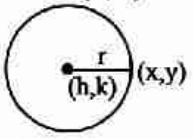
(b) coefficient of x^2 = coefficient of y^2 or $a = b \neq 0$

(c) coefficient of $xy = 0$ or $h = 0$

2. Standard Forms of Circle

(i) Central Form

(a) The equation of a circle having centre (h, k) and radius r, is $(x - h)^2 + (y - k)^2 = r^2$



(b) If the centre is origin then the equation of a circle is $x^2 + y^2 = r^2$

(ii) Diameter Form :

If (x_1, y_1) and (x_2, y_2) are the end points of a diameter of the circle, then the equation of circle is $(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

Noet :

➢ This will be the circle of least radius passing through (x_1, y_1) and (x_2, y_2)

(iii) The Parametric Equations of a Circle :

(a) The parametric equations of a circle $x^2 + y^2 = r^2$ are $x = r\cos\theta$, $y = r\sin\theta$.

(b) The parametric equations of the circle $(x-h)^2 + (y-k)^2 = r^2$ are $x = h + r\cos\theta$, $y = k + r\sin\theta$.

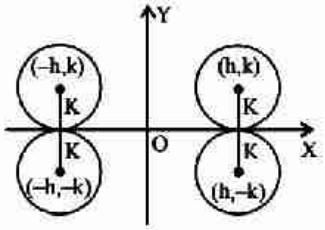
(c) Parametric equations of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ are $x = -g + \sqrt{g^2 + f^2 - c}\ \cos\theta$,

$y = -f + \sqrt{g^2 + f^2 - c}\ \sin\theta$ Where θ is the parameter and $\theta \in [0, 2\pi)$

3. Equation of Circle in Some Special Cases

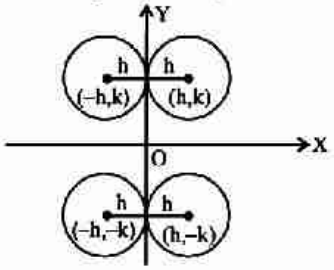
Centre of circle is (h, k)

(i) If the circle touches x axis then its equation is (Four cases)



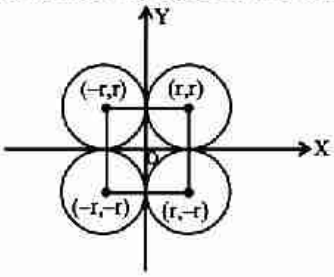
$(x \pm h)^2 + (y \pm k)^2 = k^2$

(ii) If the circle touches y axis then its equation is (Four cases)



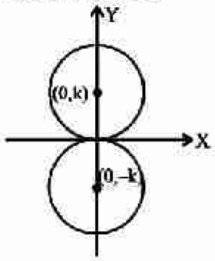
$(x \pm h)^2 + (y \pm k)^2 = h^2$

(iii) If the circle touches both the axis then its equation is (Four cases)



$(x \pm r)^2 + (y \pm r)^2 = r^2$

(iv) If the circle touches x axis at origin (Two cases)



$x^2 + (y \pm k)^2 = k^2 \Rightarrow x^2 + y^2 \pm 2ky = 0$

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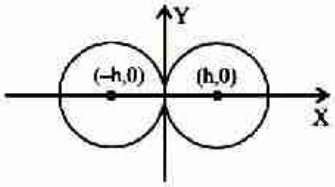
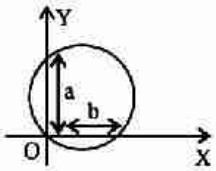
(v) If the circle touches y axis at origin (Two cases)



$(x \pm h)^2 + y^2 = h^2 \quad \Rightarrow \quad x^2 + y^2 \pm 2xh = 0$

(vi) If the circle passes through origin and cut intercept of a and b on axes, the equation of circle is (Four cases)



$x^2 + y^2 - ax - by = 0$

4. Position of a Point with Respect to a Circle

A point (x_1, y_1) lies outside, on or inside a circle

$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ according as

$S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$ is positive, zero or negative i.e.

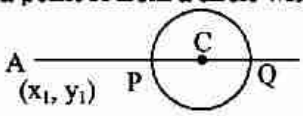
$S_1 > 0 \Rightarrow$ Point is outside the circle.

$S_1 = 0 \Rightarrow$ Point is on the circle.

$S_1 < 0 \Rightarrow$ Point is inside the circle.

5. The Least and Greatest Distance of a Point From a Circle

The greatest & the least distance of a point A from a circle with centre C & radius r is.



AQ = AC + r (greatest distance)

AP = AC – r (least distance)

6. Intercepts made on Coordinate axes by the circle

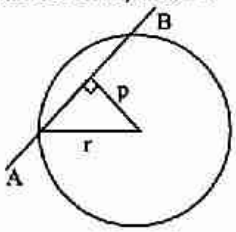
The intercept made by the circle

$x^2 + y^2 + 2gx + 2fy + c = 0$ on -

(i) x axis = $2\sqrt{g^2 - c}$ (ii) y axis = $2\sqrt{f^2 - c}$

7. Line and Circle

Let L = 0 be a line and S = 0 be a circle, if 'r' be the radius of a circle and p be the length of perpendicular from the centre of circle on the line, then if



$p > r \Rightarrow$ Line is outside the circle

$p = r \Rightarrow$ Line touches the circle (tangent)

$p < r \Rightarrow$ Line is the chord of circle (secant)

$p = 0 \Rightarrow$ Line is diameter of circle

Note :

- Length of the intercept made by the circle on the line is $= 2\sqrt{r^2 - p^2}$
- The length of the intercept made by line $y = mx + c$ with the circle $x^2 + y^2 = a^2$ is $2\sqrt{\frac{a^2(1+m^2) - c^2}{1+m^2}}$

8. Condition of Tangency

A line $L = 0$ touches the circle $S = 0$, if length of perpendicular drawn from the centre of the circle to the line is equal to radius of the circle i.e. $p = r$.

Circle $x^2 + y^2 = a^2$ will touch the line $y = mx + c$ if $\boxed{c = \pm a\sqrt{1+m^2}}$

9. Equation of Tangent

(i) Point Form (T = 0) :

(a) The equation of tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at a point (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

(b) The equation of tangent to circle $x^2 + y^2 = a^2$ at point (x_1, y_1) is $xx_1 + yy_1 = a^2$

Note :

- To get the equation of tangent at the point (x_1y_1) on any curve we replace xx_1 in place of x^2, yy_1 in place of y^2, $\frac{x + x_1}{2}$ in place of x, $\frac{y + y_1}{2}$ in place of y, $\frac{xy_1 + yx_1}{2}$ in place of xy.

(ii) Slope Form :

(a) The equation of tangent of slope m to the circle $x^2 + y^2 = a^2$ are $y = mx \pm a\sqrt{1+m^2}$ and its point of contact is $\left(\frac{\mp am}{\sqrt{1+m^2}}, \frac{\pm a}{\sqrt{1+m^2}}\right)$.

(b) The equation of tangent of slope m to the circle $(x - h)^2 + (y - k)^2 = a^2$ are $y - k = m(x - h) \pm a\sqrt{1+m^2}$ and its point of contact is $\left(h \pm \frac{ma}{\sqrt{1+m^2}}, a \mp \frac{a}{\sqrt{1+m^2}}\right)$.

(iii) Parameter Form :

The equation of tangent to the circle $(x - h)^2 + (y - k)^2 = a^2$ at the point $(h + r\cos\theta, k + r\sin\theta)$ is $(x - h)\cos\theta + (y - k)\sin\theta = a$

10. Equation of Normal

(i) Point form :

(a) The equation of normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at any point (x_1, y_1) is $y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1)$

(b) The equation of normal to the circle $x^2 + y^2 = a^2$ at any point (x_1, y_1) is $xy_1 - x_1y = 0$

(ii) Slope form :

The equation of a normal of slope m to the circle $x^2+y^2=a^2$ is $my=-x\pm a\sqrt{1+m^2}$.

(iii) Parametric Form :

The equation of normal to the circle $x^2+y^2=a^2$ at the point $(a\cos\theta, a\sin\theta)$ is $\boxed{y=x\tan\theta.}$

Note :

➢ Normal always passes through the centre of the circle.

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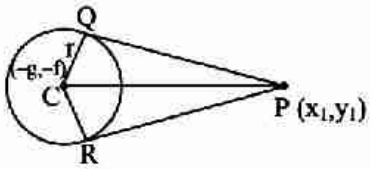
11. Length of Tangent

The length of the tangent drawn from a point $P(x_1,y_1)$ to the circle $S=x^2+y^2+2gx+2fy+c=0$ is given by

$PQ=PR=\sqrt{x_1^2+y_1^2+2gx_1+2fy_1+c}$

Length of tangent $=\sqrt{S_1}$



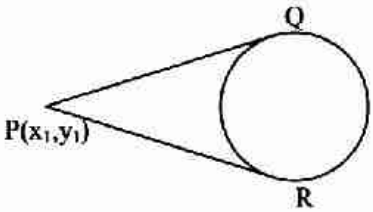
(i) If PQ is a length of the tangent from a point P to a given circle. Then PQ^2 is called the power of the point with respect to a given circle.

(ii) Area of quadrilateral PQCR $=r\sqrt{S_1}$ and angle between tangents PQ and PR is $\theta=2\tan^{-1}\dfrac{r}{\sqrt{S_1}}$

12. Pair of Tangents

From a given point $P(x_1, y_1)$, two tangents PQ and PR can be drawn to the circle $S=x^2+y^2+2gx+2fy+c=0$.

Their combined equation is $SS_1=T^2$

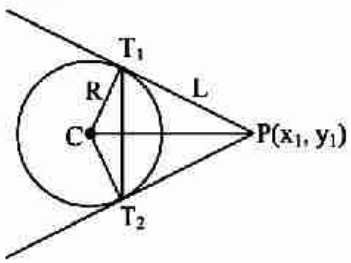


13. Chord of Contact

A line joining the two points of contacts of two tangents drawn from a point out side the circle, is called chord of contact of that point.

The equation of the chord of contact is $T=0$

i.e. $xx_1+yy_1+g(x+x_1)+f(y+y_1)+c=0$



(i) Length of chord of contact

$$T_1T_2 = \frac{2LR}{\sqrt{R^2 + L^2}}$$

(ii) Area of the triangle formed by the pair of the tangents & its chord of contact = $\frac{RL^3}{R^2 + L^2}$, where R is the radius of the circle & L is the length of the tangent from (x_1, y_1) on $S = 0$.

14. Equation of a chord whose middle point is given

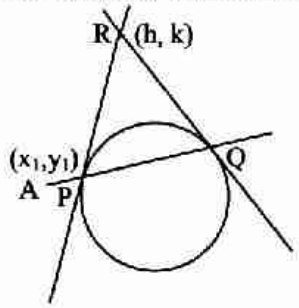
The equation of code of circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ bisected at the point (x_1, y_1) is given by $T = S_1$.
i.e. $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

15. Director Circle

The locus of the point of intersection of two perpendicular tangents to a circle is called the Director circle. Let the circle be $x^2 + y^2 = a^2$, the equation of director circle is $x^2 + y^2 = 2a^2$, director circle is a concentric circle whose radius is $\sqrt{2}$ times the radius of the given circle.

16. Pole and Polar

Let any straight line through the given point $A(x_1, y_1)$ intersect the given circle $S = 0$ in two points P and Q and if the tangent of the circle at P and Q meet at the point R then locus of point R is called polar of the point A and point A is called the pole, with respect to the given circle.



(i) The equation of the polar is the $T = 0$, so the polar of point (x_1, y_1) w.r.t. circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$

(ii) Pole of polar $Ax + By + C = 0$ with respect to circle $x^2 + y^2 = a^2$ is $\left(-\frac{Aa^2}{C}, -\frac{Ba^2}{C}\right)$

(iii) Conjugate Points : Two points A and B are conjugate points with respect to given circle, if each lies on the polar of the other with respect to the circle.

(iv) Conjugate Lines : If two lines be such that the pole of one lies on the other, then they are called conjugate lines with respect to the given circle.

17. Family of Circles

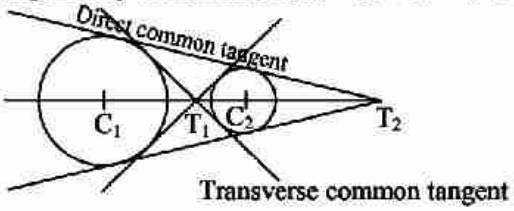
(i) The equation of the family of circles passing through the points of intersection of two circles $S_1 = 0$ & $S_2 = 0$ is $S_1 + KS_2 = 0$ $(K \neq -1)$

(ii) The equation of the family of circles passing through the point of intersection of a circle $S = 0$ & a line $L = 0$ is given by $S + KL = 0$.

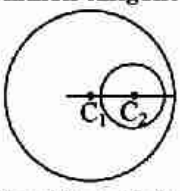
18. Relative position of two circles and No. of common Tangents

Let C_1 (h_1, k_1) and C_2 (h_2, k_2) be the centre of two circle and r_1, r_2 be their radius and C_1C_2 is the distance between their centres then

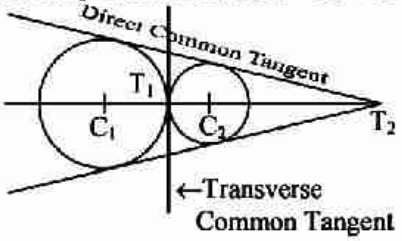
(i) When two circles are separately Condition, $C_1C_2 > r_1 + r_2$ $\Rightarrow$ 4 common tangents



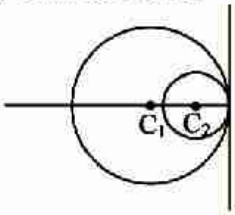
(ii) When one circle completely lie inside the other circle
Condition, $C_1C_2 < |r_1 - r_2|$ $\Rightarrow$ 0 common tangent



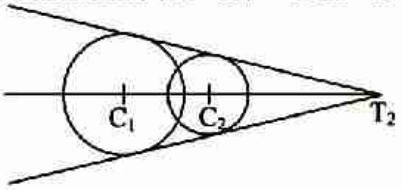
(iii) When two circles touch externally Condition, $C_1C_2 = r_1 + r_2$ $\Rightarrow$ 3 common tangents



(iv) When two circles touch internally Condition, $C_1C_2 = |r_1 - r_2|$ $\Rightarrow$ 1 common tangent



(v) When two circles intersect Conditions, $|r_1 - r_2| < C_1C_2 < r_1 + r_2$ $\Rightarrow$ 2 common tangents



(vi) **Points of intersection of common tangents:** The points T_1 and T_2 (points of intersection of transverse and direct common tangents) divides C_1C_2 internally and externally in the ratio $r_1 : r_2$.

(vii) **Equation of the common tangents at point of contact :** $S_1 - S_2 = 0$.

(viii) **Point of contact :** The point of contact divides C_1C_2 in the ratio $r_1 : r_2$ internally or externally as the case may be.

(ix) **Length of direct common tangent**

$$= \sqrt{(C_1C_2)^2 - (r_1 - r_2)^2}$$

(x) **Length of transverse common tangent**

$$= \sqrt{(C_1C_2)^2 - (r_1 + r_2)^2}$$

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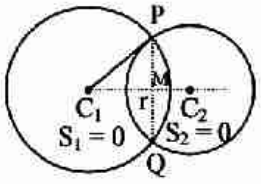
19. Equation of common chord

The chord joining the points of intersection of two given circles is known as **Common chord**.
The equation of the common chord of two circles $S_1 = 0$ and $S_2 = 0$ is $S_1 - S_2 = 0$

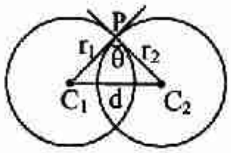


and the length of the common chord $PQ = 2\,PM = 2\sqrt{C_1P^2 - C_1M^2}$

Where C_1P = radius of the circle $S_1 = 0$ and C_1M = length of the $\perp$ from centre C_1 to the common chord PQ.

20. The Angle of Intersection of two Circles :

The angle between the tangents of two circles at the point of intersection of the two circles is called angle of intersection of two circles



$$\cos\theta = \left(\frac{r_1^2 + r_2^2 - d^2}{2r_1r_2}\right)$$

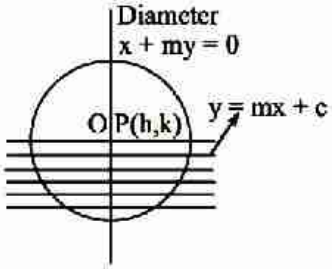
Here r_1 and r_2 are the radii of the circles and d is the distance between their centres.

20. Orthogonal Circles

Two circles are to be intersect orthogonally, if their angle of intersection is a right angle, if two circles
$S_1 = x^2 + y^2 + 2g_1x + 2f_1y + c_1 = 0$ and
$S_2 = x^2 + y^2 + 2g_2x + 2f_2y + c_2 = 0$ are orthogonal, then
$2g_1g_2 + 2f_1f_2 = c_1 + c_2$

21. Diameter of a Circle

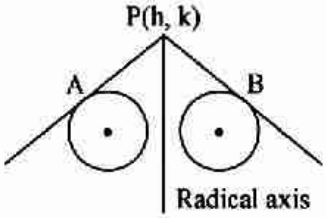
The locus of middle points of a system of parallel chords of a circle is called the diameter of a circle.



The diameter of the circle $x^2 + y^2 = r^2$ corresponding to the system of parallel chord $y = mx + c$ is $x + my = 0$

22. Radical Axis

(i) **Definition :** The locus of a point, which moves in such a way that the length of tangents drawn from it to the circles are equal and is called the radical axis.



(ii) $S_1 = 0$ and $S_2 = 0$ are two circles then equation of radical axes is $S_1 - S_2 = 0$

(iii) If circles touch each other then Radical axis is the common tangent to both the circles.

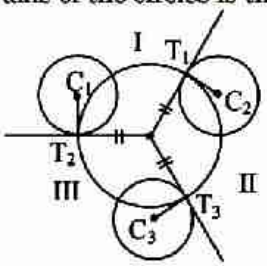
(iv) When the two circles intersect on real point then common chord is the Radical axis of the two circles.

(v) The Radical axis of the two circles is perpendicular to the line joining the centre of two circles but not always pass through mid point of it.

(vi) If two circles are orthogonal to the third circle then Radical axis of both circle passes through the centre of the third circle.

23. Radical centre

The radical centre of three circles is the point from which length of tangents on three circles are equal i.e. the point of intersection of radical axis of the circles is the radical centre of the circles.



(i) If $S_1 = 0$, $S_2 = 0$ and $S_3 = 0$ be any three given circles then the radical centre can be obtained by solving any two of the following equations
$S_1 - S_2 = 0$, $S_2 - S_3 = 0$, $S_3 - S_1 = 0$.

(ii) The circle with centre as radical centre and radius equal to the length of tangent from radical centre to any of the circle, will cut the three circles orthogonally.

24. Co-axial System of Circles

A system of circle is said to be coaxial system of circles, if every pair of the circles in the system has same radical axis.

(i) The equation of a system of coaxial circles, when the equation of the radical axis $P = \ell x + my + n = 0$ and one of the circle of the system $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, is $S + \lambda P = 0$. where λ is an arbitrary constant.

(ii) Since, the lines joining the centres of two circles is perpendicular to their radical axis. Therefore, the centres of all circles of a coaxial system lie on a straight line, which is perpendicular to the common radical axis.

25. Limiting Points

Limiting points of a system of coaxial circles are the centres of the point circles belonging to the family.

Let equation of circle b $x^2 + y^2 + 2gx + c = 0$

$\therefore$ Radius of circle = $\sqrt{g^2 - c}$

For limiting point, $r = 0$ $\quad \therefore \sqrt{g^2 - c} = 0 \Rightarrow g = \pm\sqrt{c}$

Thus, limiting points, of the given coaxial system as $(\sqrt{c}, 0)$ and $(-\sqrt{c}, 0)$.

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SHORT TRICKS

Trick -1 The x co-ordinates of two points A and B are roots of equation $px^2 + qx + r = 0$ and y co-ordinates are roots of equation $ay^2 + by + c = 0$, then to find the equation of circle with AB as a diameter in both the quadratic equation first makes the coefficient of x^2 and y^2 equal to one and then add.

$$\left(x^2 + \frac{q}{p}x + \frac{r}{p}\right) + \left(y^2 + \frac{b}{a}y + \frac{c}{a}\right) = 0$$

$$\Rightarrow x^2 + y^2 + \frac{q}{p}x + \frac{b}{a}y + \frac{r}{p} + \frac{c}{a} = 0$$

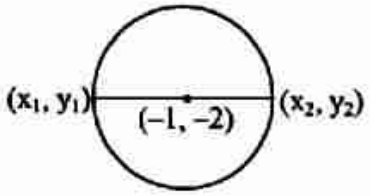
Q.1 The x coordinates of two points A and B are roots of equation $x^2 + 2x - a^2 = 0$ and y coordinate are roots of equation $y^2 + 4y - b^2 = 0$ then equation of the circle which has diameter AB is-

(A) $(x-1)^2 + (y-2)^2 = 5 + a^2 + b^2$ (B) $(x+1)^2 + (y+2)^2 = \sqrt{(5+a^2+b^2)}$

(C) $(x+1)^2 + (y+2)^2 = (a^2+b^2)$ (D) $(x+1)^2 + (y+2)^2 = 5 + a^2 + b^2$

Sol. [D]

Proper Method



$x_1 + x_2 = -2$

$x_1x_2 = -a^2$

$y_1 + y_2 = -4$

$y_1y_2 = -b^2$

Equation of circle

$(x+1)^2 + (y+2)^2 = [(x_1 - x_2)^2 + (y_1 - y_2)^2]/4$

$= [(x_1 + x_2)^2 - 4x_1x_2 + (y_1 + y_2)^2 - 4y_1y_2]/4$

$= [4 + 4a^2 + 16 + 4b^2]/4$

$= 4[5 + a^2 + b^2]/4$

$= 5 + a^2 + b^2$

Short Trick

Equation of circle

$(x^2 + 2x - a^2) + (y^2 + 4y - b^2) = 0$

$(x+1)^2 + (y+2)^2 = 5 + a^2 + b^2$

Q.2 The abscissa of two points A and B are the roots of the equation $x^2 + 2ax - b^2 = 0$ and their ordinates are the roots of the equation $y^2 + 2py - q^2 = 0$. The radius of the circle with AB as a diameter will be -

(A) $\sqrt{a^2+b^2+p^2+q^2}$ (B) $\sqrt{b^2+q^2}$ (C) $\sqrt{a^2+b^2-p^2-q^2}$ (D) $\sqrt{a^2+p^2}$

Sol. [A]

Proper Method

Let A = (α, β); B = (γ, δ). Then

$\alpha + \gamma = -2a$, $\alpha\gamma = -b^2$

and $\beta + \delta = -2p$, $\beta\delta = -q^2$

Now equation of the required circle is

$(x-\alpha)(x-\gamma) + (y-\beta)(y-\delta) = 0$

$\Rightarrow x^2 + y^2 - (\alpha+\gamma)x - (\beta+\delta) + \alpha\gamma + \beta\delta = 0$

$\Rightarrow x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

Its radius = $\sqrt{a^2+b^2+p^2+q^2}$

Short Trick

Equation of circle

$x^2 + y^2 + 2ax + 2py - b^2 - q^2 = 0$

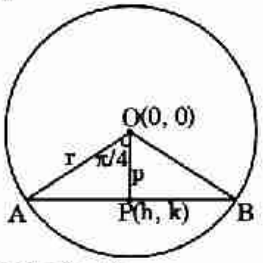
C(–a, – p), r = $\sqrt{a^2+p^2+b^2+q^2}$

Trick -2 The locus of mid point of chord of circle $(x-h)^2+(y-k)^2=r^2$ which subtends an angle θ at centre of circle is $(x-h)^2+(y-k)^2=r^2\cos^2\frac{\theta}{2}$

Q.3 Find the locus of mid point of chords of circle $x^2+y^2=25$ which subtends right angle at origin-
(A) $x^2+y^2=25/4$ (B) $x^2+y^2=5$ (C) $x^2+y^2=25/2$ (D) $x^2+y^2=5/2$

Sol. **[C]**

Proper Method



Let the mid point be (α, β)
In ΔOAP

$\cos 45^\circ = \frac{p}{r}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sqrt{h^2+k^2}}{5}$

$\Rightarrow h^2+k^2=\frac{25}{2}$

$\therefore$ locus $x^2+y^2=\frac{25}{2}$

Short Trick

$x^2+y^2=25\cos^2 45^\circ$

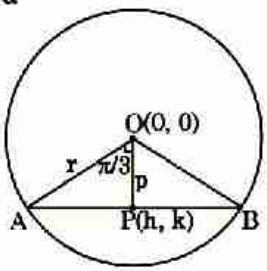
$x^2+y^2=\frac{25}{2}$

Q.4 Let C be the circle with centre (0, 0) and radius 3 units. The equation of the locus of the mid points of the chords of the circle C that subtend an angle of $\frac{2\pi}{3}$ at its centre is – **[AIEEE-2006]**

(A) $x^2+y^2=1$ (B) $x^2+y^2=\frac{27}{4}$ (C) $x^2+y^2=\frac{9}{4}$ (D) $x^2+y^2=\frac{3}{2}$

Sol. **[C]**

Proper Method



In ΔOAP

$\cos\frac{\pi}{3}=\frac{p}{r}$

$\frac{1}{2}=\frac{\sqrt{h^2+k^2}}{3}$

$\Rightarrow h^2+k^2=\frac{9}{4}$

$\therefore$ locus $x^2+y^2=\frac{9}{4}$

Short Trick

$x^2+y^2=9\cos^2\frac{\pi}{3}$

$x^2+y^2=\frac{9}{4}$

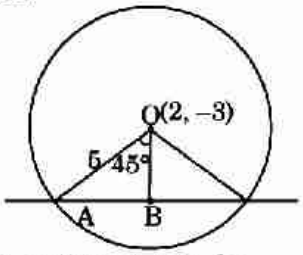
Q.5 Find the locus of mid point of chord of circle $x^2 + y^2 - 4x + 6y - 12$ which subtends a right angle at centre of the circle.

(A) $(x-2)^2 + (y+3)^2 = \frac{25}{2}$ (B) $(x+2)^2 + (y-3)^2 = \frac{25}{2}$

(C) $(x-2)^2 + (y-3)^2 = \frac{25}{2}$ (D) $(x+2)^2 + (y+3)^2 = \frac{25}{2}$

Sol. **[A]**

Proper Method



Let the mid point of chord is (h, k).

In ΔOAB $\cos 45^\circ = \frac{OB}{5}$

$\Rightarrow OB = 5\cos 45^\circ$

$\Rightarrow \sqrt{(h-2)^2 + (k+3)^2} = 5\frac{1}{\sqrt{2}}$

$\Rightarrow (h-2)^2 + (k+3)^2 = \frac{25}{2}$

$\therefore$ locus $(x-2)^2 + (y+3)^2 = \frac{25}{2}$

Short Trick

$(x-h)^2 + (y-k)^2 = r^2\cos^2\frac{\theta}{2}$

$\Rightarrow (x-2)^2 + (y+3)^2 = \frac{25}{2}$

Trick -3 Method of substitution

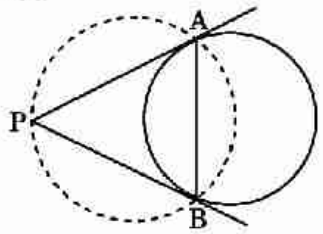
Q.6 Tangents drawn from the point P(1, 8) to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is - **[IIT-2009]**

(A) $x^2 + y^2 + 4x - 6y + 19 = 0$ (B) $x^2 + y^2 - 4x - 10y + 19 = 0$

(C) $x^2 + y^2 - 2x + 6y - 29 = 0$ (D) $x^2 + y^2 - 6x - 4y + 19 = 0$

Sol. **[B]**

Proper Method



Equation of line AB is (chord of contact)

$x + 8y - 3(x+1) - 2(y+8) - 11 = 0$

$\Rightarrow -2x + 6y - 30 = 0$

$\Rightarrow x - 3y + 15 = 0$

Equation of circle passes through A & B is given by

$(x^2 + y^2 - 6x - 4y - 11) + \lambda(x - 3y + 15) = 0$

As, (3, 2) will lie on it $-24 + 12\lambda = 0$

$\lambda = 2$

Hence equation is $x^2 + y^2 - 4x - 10y + 19 = 0$

Short Trick

As the circumcircle passes through the point (1, 8), so this point will satisfy equation of circle. Hence (B) is correct option.

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Q.7 The equations of the tangents drawn from the point (0, 1) to the circle $x^2 + y^2 - 2x + 4y = 0$ are-

(A) $2x - y + 1 = 0,\ x + 2y - 2 = 0$ (B) $2x - y - 1 = 0,\ x + 2y - 2 = 0$

(C) $2x - y + 1 = 0,\ x + 2y + 2 = 0$ (D) $2x - y - 1 = 0,\ x + 2y + 2 = 0$

Sol. **[A]**

Proper Method	**Short Trick**
$T^2 = SS_1$ $(y - x + 2(y + 1))^2 = (x^2 + y^2 - 2x + 4y)(5)$ $\Rightarrow (3y - x + 2)^2 = 5x^2 + 5y^2 - 10x + 20y$ $\Rightarrow 9y^2 + x^2 + 4 - 6xy - 4x + 12y$ $= 5x^2 + 5y^2 - 10x + 20y$ $\Rightarrow 4x^2 - 4y^2 + 6xy - 6x + 8y - 4 = 0$ $\Rightarrow 2x^2 - 2y^2 + 3xy - 3x + 4y - 2 = 0$ $\Rightarrow (2x - y + 1)(x + 2y - 2) = 0$	(0, 1) satisfy both the tangents Hence (A) is correct option.

Q.8 The equation of the normal to the circle $x^2 + y^2 - 8x - 2y + 12 = 0$ at the points whose ordinate is -1, will be-

(A) $2x - y - 7 = 0,\ 2x + y - 9 = 0$ (B) $2x + y - 7 = 0,\ 2x + y + 9 = 0$

(C) $2x + y + 7 = 0,\ 2x + y + 9 = 0$ (D) $2x - y + 7 = 0,\ 2x - y + 9 = 0$

Sol. **[A]**

Proper Method	**Short Trick**
As ordinate is -1 $x^2 - 8x + 15 = 0 \Rightarrow x = 3, 5$ that is points are $(3, -1)$ & $(5, -1)$ At (3, 1), equation of normal $y + 1 = \left(\frac{-2}{-1}\right)(x - 3)$ $\Rightarrow 2x - y - 7 = 0$ At $(5, -1)$ $y + 1 = \left(\frac{-2}{1}\right)(x - 5)$ $\Rightarrow 2x + y = 9$	Normal of the circle passes through centre of circle Hence, centre (4, 1) satisfy the option (A)

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